# STUDY OF SUPERSONIC TWIN JET COUPLING USING HIGHER ORDER SPECTRAL ANALYSIS

K. Srinivasan[*], Praveen Panickar[†], Ganesh Raman[‡], Byung-Hun Kim[§], David Williams[¶]
Fluid Dynamics Research Center
Illinois Institute of Technology
10 West 32nd Street
Chicago, IL – 60616
Phone: (312)-567-3554
Fax: (312)-567-7230
Email: Raman@iit.edu

## Abstract

The purpose of this paper is to understand non-linear processes occurring during the coupling of twin jet plumes using higher order spectral methods. To the best of our knowledge, most previous studies have used linear spectral analysis to document coupling and have focused on identifying configurations and conditions that produce coupling, without much knowledge of the 'why' and 'how.' Our work demonstrates the inadequacies of linear spectral analysis when closely spaced multiple screech sources exist in complex configurations. We use the cross-bicoherence for identifying the non-linear interactions. In order to evaluate the efficacy of the tool in identifying non-linear coupling we chose rectangular nozzles with spanwise oblique exit geometries. Jets from such nozzles are known to be rich in closely spaced complex screech source structures. In addition, they can produce multiple screech tones with the simultaneous presence of multiple feedback loops. The twin jets could be placed in two configurations, one that produced a 'V' shape in the inter-nozzle region and another that produced an 'arrow head' shape. The following significant findings emerged from our study: (i) some configurations that were apparently uncoupled by linear spectral analysis metrics were found to be non-linearly coupled. (ii) two types of non-linear coupling were observed – one dominated by the fundamental and its interaction with higher modes, and another that displayed clusters of interactions between a frequency component and its sidebands. (iii) a new interaction density metric was developed to quantify non-linear coupling. (iv) a second metric known as the average interaction density was shown to increase sharply during coupling mode transition. Our results indicate that nonlinear spectral analysis has the potential to uncover twin jet coupling mysteries that have eluded researchers for many years.

## Introduction

### *Background*

The acoustic interaction between two proximal jet plumes has been investigated in this work. Such interactions could alter the flow and acoustic field substantially, leading to enhanced acoustic pressures in the near-field. Acoustic fields have traditionally been characterized by second order spectral analyses. These methods are sufficient to describe acoustic fields with a single acoustic source, or, in the case of multiple sources where the spatial separation of acoustic sources is much larger than the characteristic acoustic wavelength. In the case of shock-containing jets, there could be multiple acoustic sources of comparable strengths spatially separated within a few acoustic wavelengths. There is evidence in the literature for jets with multiple screech tones, with their corresponding feedback loops. In such circumstances, the interaction between waves occurs much earlier than they attain sphericity or planarity, thereby making the interactions non-linear. When such complexities are possible in a single jet plume, further complexity is inevitable when the shock-cells are spanwise oblique, and when another such plume is located in close vicinity.

---

[*] Visiting Assistant Professor (Parent Institution: IIT Guwahati, India), Member, AIAA
[†] Graduate Student, Student Member, AIAA
[‡] Associate Professor, Associate Fellow, AIAA (author for correspondence)
[§] Graduate Student, Student Member, AIAA
[¶] Professor, Associate Fellow, AIAA

The present study focuses on the acoustic interactions in such jets with complex shock structures. The subsequent section briefly reviews the literature concerning (a) the acoustics of nozzles with complex geometry and (b) instances of other flow situations where higher order spectral analyses have been successfully used. These two discussions in conjunction, weave the background, motivation and methodology for the present work.

### *Review of Relevant Literature*

Interest in acoustic modes and their coupling in jet flows arose following observations of their undesirable consequences like acoustic fatigue damage in laboratory and full-scale tests. Early observations were made in circular jets in single and twin configurations. Later, the focus diversified to rectangular and other non-axisymmetric configurations owing to their aerodynamic, acoustic and stealth benefits. Since the present paper focuses on flows from rectangular nozzles, the literature discussion is being restricted to studies on rectangular configurations.

Screeching jets from uniform and spanwise oblique rectangular nozzles have been studied in great detail by several researchers[1-6]. Raman and Taghavi[2] conducted a detailed study of the near acoustic field and the coupling mechanism of twin rectangular supersonic jets having uniform exit geometry. They found that there were two modes of coupling that prevailed - the symmetric mode that augmented the screech amplitude and the antisymmetric mode that suppressed it and both these modes were mutually exclusive. A companion study by Taghavi and Raman[3] on twin jets having straight rectangular exit geometry in various configurations found that the shock spacing did not change significantly when the jets coupled. The coupling of twin supersonic jets of double beveled exit geometry was studied by Raman[4], who found that twin double beveled jets can couple and may lead to either an augmentation or suppression of sound in the inter-nozzle region depending on the fully expanded Mach number at which the jets were operating. Although previous work has illuminated some aspects of individual single beveled nozzles[5,6], to the best of our knowledge there is no published information on the interaction of twin supersonic jets having single beveled exit geometry.

The authors recently conducted experiments on such configurations using conventional second order spectral analyses, from which significant information of scientific and engineering value emerged, and key results were presented at an earlier conference[7]. However, since the present study follows the earlier work, some important results are summarized below. The authors' earlier work revealed that while the single beveled jet exhibits spanwise symmetric, spanwise antisymmetric, and spanwise oblique modes while operating individually (Figure 1(a)), they coupled only in spanwise symmetric and spanwise antisymmetric modes when operated in the twin jet mode. Further, the coupling in the twin jet was restricted to a configuration wherein the jets formed a V-shaped inter-nozzle region as shown in Figure 1(b). The other possible twin jet configuration forming an arrowhead inter-nozzle region (Figure 1(c)) did not show coupling behavior. These results were obtained using power spectra, spanwise phase measurements and directivity measurements.

Based on these interesting findings, and intrigued by the non-linearity in the coupling phenomenon, the authors felt the need for further analysis of these flows using higher order spectral methods. Elementary non-linear spectral methods use triple correlations. The auto-bicoherence and the cross-bicoherence are the non-linear analogues of the auto-spectrum and cross-spectrum functions in the conventional second-order spectral analyses. Thomas and Chu[8] studied the evolution of a planar shear layer using non-linear spectral analyses. The higher order tool used was shown successful in describing the non-linear evolution characteristics of the shear layer. Walker and Thomas[9] have used third order spectral analysis to quantify the quadratic interactions in a screeching rectangular jet. Their work demonstrated the ability of higher order methods in identification of quadratic interactions. The above two references contain numerous references relevant to the use of non-linear spectral analyses in the study of shear flows. The focus of this paper is described below.

### *Present Work*

The focus of this paper is to study single and twin rectangular jets of complex geometry, and study their coupling behavior in terms of higher order properties like cross-bicoherence. Therefore our objectives are to quantify the nonlinear interactions occurring in these jets, and explore the possibility of tracing the evolution of power spectra, and explain the behavior in these complex flow systems, hitherto unknown.

To accomplish our objectives, three basic configurations were chosen: (i) V-shaped twin jets, (ii) Arrowhead shaped twin jet, and (iii) single jet. The dimensions of individual nozzles were 35.56 mm in the spanwise direction and 5.08 mm in the transverse direction. The fully expanded Mach number range covered in the study was 1.29 ≤ $M_j$ ≤ 1.51, beyond which the jets did not screech. The inter-nozzle spacing was varied in the V-shaped twin jet configuration in the range 7.3 ≤ *s/h* ≤ 7.9, where *s* and *h* are defined in figure 1. Inter-nozzle spacing was not varied in the arrowhead configuration since it did not show coupling. The time series data from two spanwise microphones were analyzed to yield the power spectra and cross-bicoherence. The experimental details are outlined in the subsequent section.

where $X^{(k)}(f)$ and $Y^{(k)}(f)$ are the DFT of discrete time series signals *x(t)* and *y(t)*. Then, an ensemble average is done to obtain the final estimate of discrete cross-bispectrum.

$$S_{YXX}(f_1, f_2) = \frac{1}{M}\sum_{k=1}^{M} S_{YXX}^{(k)}(f_1, f_2) \tag{2}$$

The cross-bicoherence spectrum is then obtained by normalizing this quantity with the power spectra of the two signals as follows:

$$b_c^2(f_1, f_2) = \frac{\left|S_{YXX}(f_1, f_2)\right|^2}{\left(\frac{1}{M}\sum_{k=1}^{M}\left|Y^{(k)}(f_1+f_2)\right|^2\right)\left(\frac{1}{M}\sum_{k=1}^{M}\left|X^{(k)}(f_1)X^{(k)}(f_2)\right|^2\right)} \tag{3}$$

## Experimental Apparatus and Procedure

The experiments were carried out in the high speed jet facility at the Illinois Institute of Technology, Chicago. The test facility details are described elsewhere[7]. All acoustic measurements were made using ¼" diameter B & K microphones, and the microphone locations are shown in Figure 1. These microphones have a flat response up to 100 kHz. All the data acquisition was achieved using a PC based National Instruments data acquisition board capable of acquiring 1.6 megasamples/second using Labview 6. The uncertainties in the present study are mainly due to frequency and sound pressure. The uncertainty in the sound pressure is within 1%, and that in the frequency is within 100 Hz. The power spectra and cross-bicoherence were computed and plotted using MATLAB 6.5. The computation of cross-bicoherence is described in the following section.

## Computation of Cross-Bicoherence

The third order quantities like bispectrum and bicoherence are obtained from the Fourier transform of the triple correlation of the time series signals. They are exactly analogous to the second order spectral methods and rules governing correlation and spectra. A detailed introduction to these methods and their use in the study of high speed jet flows has been presented by Thomas[10]. Therefore, only the mathematical expression for cross-bicoherence is given below. Cross-bicoherence is the normalized cross-bispectrum. The discrete cross-bispectrum is expressed for an ensemble as,

$$S_{YXX}^{(k)}(f_1, f_2) = Y^{(k)}(f_1+f_2)\, X^{(k)*}(f_1) X^{(k)*}(f_2) \tag{1}$$

The computation of these quantities is simplified by using symmetry properties in the frequency domain. Therefore, referring to Figure 2(b), only the region bounded by the two $45^0$ lines in the upper half plane, and that in the lower half plane are unique. Exploiting these observations resulting from the symmetry properties, the computation is restricted to this region. The computation is done in MATLAB 6.5. An example demonstrating the use of cross-bicoherence is presented in the following paragraph.

Sum and difference interactions between signals and the ability of cross-bicoherence to detect them can be illustrated by generating two sinusoids as follows:

$$f(t) = \sin w_1 t + \sin w_2 t\,, \tag{4}$$

and

$$g(t) = \sin w_1 t \, \sin(w_2 t + e) \tag{5}$$

where, $\omega_1$ and $\omega_2$ are the circular frequencies, and ε is a small phase difference added to one of the sinusoids. These two signals are used as inputs to the program to compute the cross-bicoherence. The power spectra of the signals are shown in Figure 2(a), and cross-bicoherence is shown in Figure 2(b). Note that in this example, the two sinusoids have frequencies of 5 kHz and 8 kHz. It can be seen that the cross-bicoherence shows two distinct peaks, at ($\omega_1$, $\omega_2$) (8kHz, 5kHz) and ($\omega_1$, -$\omega_2$), corresponding to the frequencies $\omega_1+\omega_2$, and $\omega_1-\omega_2$ in the modulated signal. Hence, the advantage of cross-bicoherence is its ability to identify such non-linear interactions, which is difficult with second order methods such as power spectra. For instance, the power spectra of sin $(\omega_1+\omega_2)t$ + sin $(\omega_1-\omega_2)t$, and $2(\sin\omega_1 t)(\sin\omega_2 t)$ would be congruent despite the differences in the time series. In essence, second order spectra obliterate phase information, which could be very important to understand mechanisms in a complex physical process as in the present case.

In the above discussions, it is implicit that the relative phase difference between the two modulated signals should be small for the cross-bicoherence to be detectable. This concept is elucidated by computing the cross-bicoherence for various values of $\varepsilon$. The variation of the peak cross-bicoherence between the above test signals and the relative phase difference between the modulated signals is plotted in Figure 3. This plot shows that cross-bicoherence is unchanged for small phase differences, but drops significantly beyond a phase difference of around $\pi/2$. This observation however, does not prevent cross-bicoherence from being a useful tool in high speed jet flows. This is because non-linear interactions take place within a small region near the shock cells, and can be considered compact, and hence the interactions can be assumed to be occurring between waves before their relative phase difference becomes large enough to diminish the cross-bicoherence. Therefore, in the present paper, lower values of cross-bicoherence are assumed to be a result of weaker interactions, or those between waves that emerge much later in the chronology of interaction events, rather than as a result of a substantial phase difference.

## Results and Discussion

The experiments resulted in time series data from the two microphones for fifteen Mach numbers, for the six geometric configurations considered in this study. For each set of time series data, the power spectrum was obtained. This resulted in 180 power spectra. Further, each pair of time series data resulted in a cross-bicoherence spectrum. Therefore, owing to the volume of the data, only a few comparisons are being presented. In order to view all the results in a common perspective, some metrics have been identified, that would be discussed later in this section. In the following paragraphs, the variation in the cross-bicoherence spectrum with change in the jet configuration is presented.

### *V-shaped and Arrowhead Configurations*

Figure 4 compares the cross-bicoherence spectra and power spectra for V-shaped and Arrowhead shaped twin jets. It is evident that non-linearity is much lower in the arrowhead configuration compared to the V-shaped case. The extent of non-linearity can be seen from the presence of dots in the cross-bicoherence spectra and the magnitude. Each peak in the cross-bicoherence spectrum denotes the non-linear interaction between the corresponding frequencies. Thus, the sparsely populated spectrum corresponding to the arrowhead configuration is indicative of lower levels of quadratic interaction between various modes. This observation is in agreement with our earlier results obtained through linear phase coherence measurements that showed that while the V-shaped configuration coupled, the arrowhead configuration did not couple at all. Another observation worthy of mention is that the power spectra of the two jets are not drastically different. In fact, the energy possessed by the peak frequency in the two jets is of the same order, and dominant compared to rest of the frequencies. Further, there are non-harmonically related frequencies in both the spectra. Thus, it is clear that presence of non-harmonically related frequencies does not necessarily produce non-linear interactions. In this context, the results concerning the phase dependence of cross-bicoherence may be recollected, wherein it was mentioned that the phase difference between interacting waves should be small to produce noticeable non-linear interactions. In the case of the arrowhead configuration, it can be expected that the sound sources in the two jets are farther apart compared to the V-shaped jet, and thus acquire a larger phase difference before any interaction could occur. It is also interesting to observe that the total power of the spectra is inconsequential to non-linear interactions. The arrowhead configuration has larger energy (as seen from the area under the power spectrum) and yet the non-linearity is much less.

### *Effect of Inter-Nozzle Spacing*

Figure 5 compares the effect of inter-nozzle spacing on the non-linear interactions in V-shaped twin jet configurations. As can be seen from the figures, the cross-bicoherence spectra show localized arrays of dots in each case. The array size grows as inter-nozzle spacing is increased. The interactions cluster around three main zones concerning: (1) interactions of the fundamental and its neighboring frequencies with lower frequencies, (2) self interaction of fundamental frequency (and its neighborhood) with themselves, (3) difference interaction between the neighborhood of the second harmonic and the neighborhood of the fundamental. There are other prominent zones, but the aforementioned zones contain high levels of cross-bicoherence. It should be pointed again that the spectra are not substantially different from one another. Therefore, it appears that an increase in inter-nozzle spacing promotes non-linearity. However, it is logical that at much larger inter-nozzle spacings ($s>>h$), the phase relationship would be lost between interacting modes leading to much lower non-linearities.

A similar comparison is shown in figure 6, for a slightly higher Mach number of 1.46. The purpose of this figure is to point out the differences in the cross-bicoherence spectra across different Mach numbers. The arrays of dots seen in figure 5 are no longer seen in this figure. This is because, there are no frequency components of comparable magnitude in the neighborhood of the fundamental frequency in the spectra. Therefore, the interaction is restricted to the self-interaction of the fundamental screech frequency. Another contrasting observation is the presence of dots (interactions) along horizontal and vertical lines at positions corresponding to the fundamental frequency. These are marked on figure 6(e) as "A" and "B". These interactions indicate that at these Mach numbers, the most preferred frequencies for non-linear interactions are the fundamental and the first harmonic.

*Most Preferred Frequency*

Further, in most of the cross-bicoherence spectra in figure 6, as well as in some others like figure 5(g), there is a trail of interaction zones tending to form a $-45^0$ line in the cross-bicoherence spectrum (marked in figure 6(g) as "C"). Such lines in the cross-bicoherence spectrum indicate that the resultant frequency corresponding to the line is the most preferred. In all these cases, the fundamental frequency seems to be the most preferred resultant frequency from the non-linear interactions. That is, the presence of a trail like pattern indicates that there are numerous interactions along the line. However, since all those interactions result in the "most preferred frequency."

*New Metrics Defined*

In order to view all the observations in a common perspective, some metrics have been attempted. The first, termed as "*Interaction density*" ($I_c$), is the number of peaks in the cross-bicoherence spectrum above a certain threshold value. In this study, threshold values of 0.3, and 0.4 have been used. The threshold is indicated in the subscripts.

Figure 7 shows the variation of interaction density with Mach number for all the configurations studied, for threshold values of 0.3 and 0.4. Notable observations are: (i) the interaction density of V-shaped twin jets are much higher compared to arrowhead shaped jet, (ii) single jet has values somewhere in the middle, and (iii) the interaction density for the V-shaped twin jets show a sharp increase around Mach number 1.4. This Mach number is the one at which a coupling shift occurred from symmetric to antisymmetric, in our earlier work[7]. Therefore, there seems to be a strong correspondence between these two observations. Further, our earlier studies had concluded that coupling existed only when the inter-nozzle spacing was $s/h = 7.3$. However, the present results show that non-linear interactions are similar at higher spacings, and in fact, tend to increase with spacing. This seems to indicate that although there is no evidence of linear coupling at higher spacings, there seems to be a strong non-linear coupling at higher spacings. One interesting observation is that this behavior of interaction density is the same for the two values of bicoherence threshold considered. This enhances our confidence in the aforementioned results.

In order to consider the effect of inter-nozzle spacing and Mach number on the interaction density, the interaction densities of V-shaped twin jets were averaged at each Mach number using four inter-nozzle spacings of $s/h = 7.3$, 7.5, 7.7, and 7.9. The average interaction densities so obtained, are plotted against Mach number as shown in figure 8(a & b). These two curves correspond to cross-bicoherence threshold values of 0.3, and 0.4, respectively. From these plots, it is clear that the non-linearity sharply increases at around a Mach number of 1.4, where linear phase coherence shows a switch from symmetric to anti-symmetric coupling. Thus, it may be conjectured that a coupling mode switch is accompanied by an increase in the extent of non-linearity. Plots 8 (c and d) show the average interaction densities, calculated by averaging the interaction densities over all Mach numbers for a certain inter-nozzle spacing. These plots show the effect of inter-nozzle spacing considering the entire Mach number range. The plots show that there is a monotonic increase in the interaction density with inter-nozzle spacing, and the observations are similar for both cross-bicoherence threshold values considered. This shows that an increase in inter-nozzle spacing promotes non-linear interactions among the sound sources. However, there should be a limiting case, since as $s/h \rightarrow \infty$, logically, the phase correspondence would be lost, and hence the interactions would lose strength, and the interaction density would decrease and tend to zero.

## Conclusions

Based on the above results, the following conclusions may be drawn:

(1) V-shaped twin jets shown to be uncoupled beyond a certain inter-nozzle spacing with second order methods, show non-linear coupling at higher spacings when higher order spectra are used.

(2) Two patterns of the cross-bicoherence spectrum were observed, one in which an array of dots dominate the spectrum, indicating interactions between close frequencies, and the other in which the dots tending to form straight lines appear in the spectrum. This case indicated the preference of the fundamental frequency as a participant in the nonlinear interactions, as well as a resultant frequency of the interactions.

(3) A new metric has been defined, termed as the "interaction density", based on the number of peaks in the cross-bicoherence spectrum, and seems to be a relevant parameter to quantify nonlinear coupling.

(4) The average interaction density increases sharply around a Mach number that showed a transition between symmetric to antisymmetric coupling in linear phase coherence studies. Therefore, modal transitions in coupling can be believed to be accompanied by a large amount of non-linear interactions.

(5) The average interaction density increases monotonically with inter-nozzle spacing for the range of spacings considered in the present study.

Our future studies will focus on how the power spectrum evolves from the non-linear interactions.

## Acknowledgements

This work was funded by the US Air Force Office of Scientific Research (AFOSR) with Dr. John Schmisseur as Program Manager.

## References

1. Tam, C.K.W., Shen, H., & Raman, G., "Screech Tones of Supersonic Jets from Beveled Rectangular Nozzles," AIAA paper 97-0143, 1997.

2. Raman, G., & Taghavi, R., "Coupling of Twin Rectangular Supersonic Jets," Journal of Fluid Mechanics, Vol. 354, Jan. 1998, pp. 123-146.

3. Taghavi, R.R., & Raman, G., "Interaction of Twin Rectangular Supersonic Jets in Various Configurations", 1998 ASME Fluids Engineering Division Summer Meeting, Washington D.C., FEDSM98-5243.

4. Raman, G., "Coupling of Twin Supersonic Jets of Complex Geometry", Journal of Aircraft, Vol. 36, No. 5, 1999, pp. 743-749.

5. Raman, G., "Screech Tones from Rectangular Nozzles with Spanwise Oblique Shock-Cell Structures", AIAA Paper 96-0643, 1996.

6. Raman, G., "Shock-Induced flow resonance in Supersonic Jets of Complex Geometry", Physics of Fluids, Vol. 11, No. 3, 1999, pp. 692-709.

7. Praveen P., Srinivasan, K., and Raman, G. "Interactions of Twin Jets from Nozzles of Spanwise Oblique Geometry", AIAA Paper 2003-1197, 2003.

8. Thomas, F.O. and Chu, H.C., "Nonlinear Wave Coupling and Subharmonic Resonance in Planar Jet Shear Layer Transition", Physics of Fluids (A), Vol. 5, No.3, March 1993, pp. 630-646.

9. Walker, S.H. and Thomas, F.O., "Experiments Characterizing Nonlinear Shear Layer Dynamics in a Supersonic Rectangular Jet Undergoing Screech," Physics of Fluids, Vol.9, pp. 2562-2579.

10. Thomas, F.O., "Applications of Advanced Data Analysis Techniques to High Speed Jets", In "High Speed Jet Flows", Editors, Morris, P.J., McLaughlin, D.K., and Raman, G., Taylor and Francis, NY, 2003 (in press).

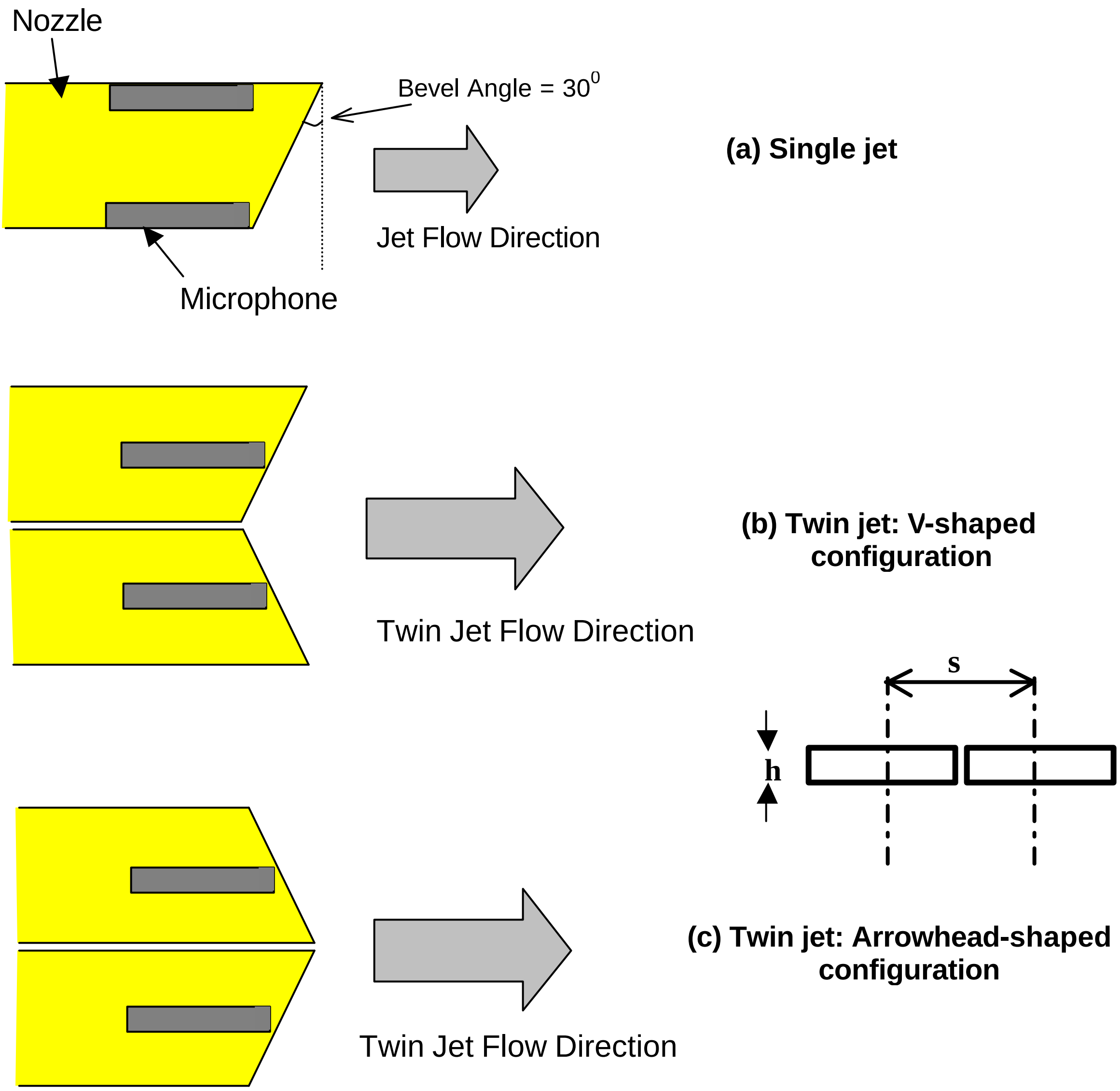


**Figure 1.** Single and twin jet configurations studied. Microphone positions are shown in each configuration.

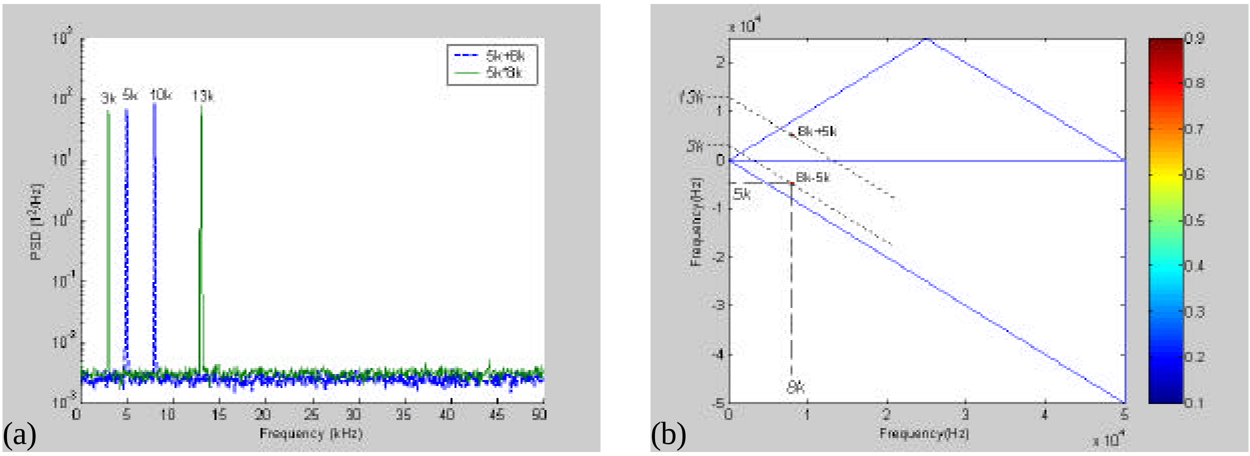


**Figure 2.** (a) Power Spectra of the test signals. (b) Cross-Bicoherence plot.

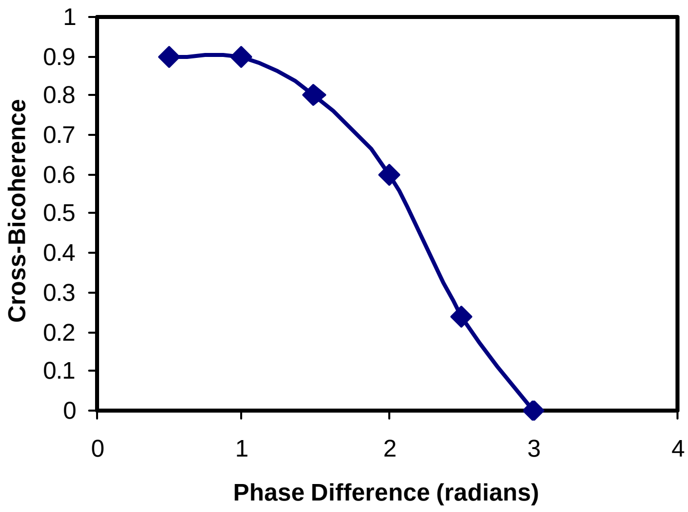
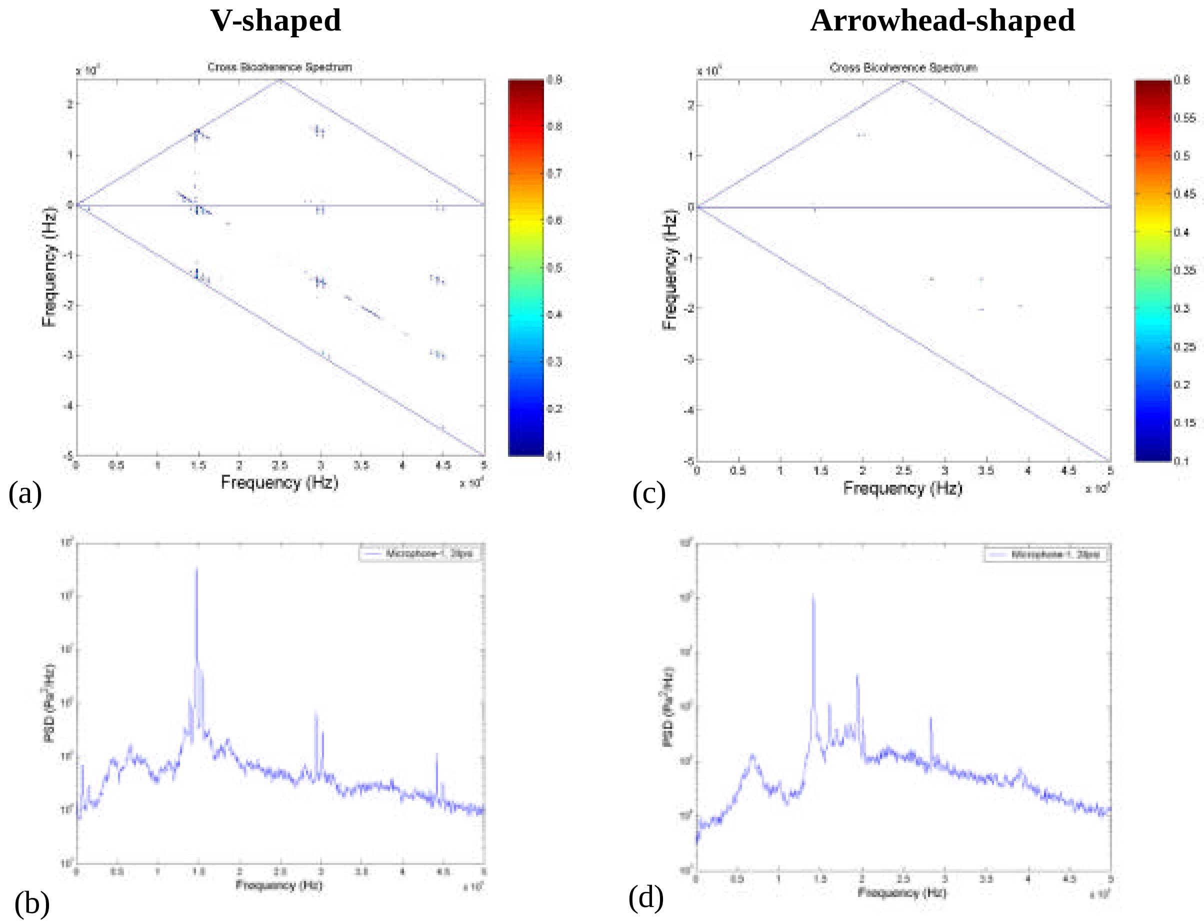


**Figure 3.** Sensitivity of cross-bicoherence to phase difference between modulated test sinusoids.


V-shaped

Arrowhead-shaped

(a)

(b)

(c)

(d)

**Figure 4.** Cross-bicoherence and spectra of twin jets at $M_j$ = 1.33, $s/h$ = 7.3 for V-shaped and Arrowhead configurations. (a) Cross-bicoherence spectrum for V-shaped, (b) Power spectrum for V-shaped, (c) Cross-bicoherence spectrum for Arrowhead shaped, (d) Power spectrum for Arrowhead.

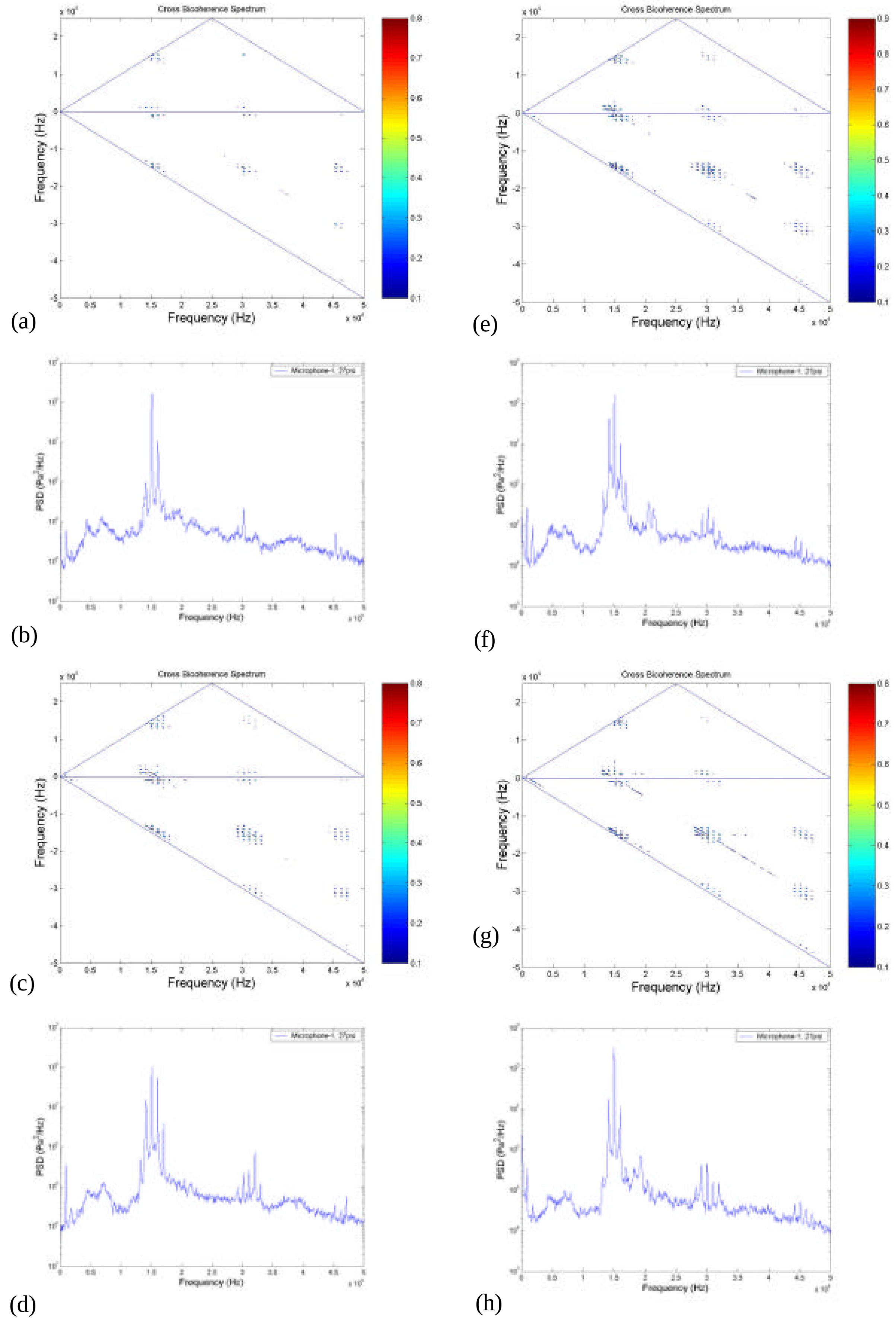


**Figure 5.** Cross-bicoherence and spectra of V-shaped twin jets at $M_j = 1.32$, for various internozzle spacings. (a,b): $s/h = 7.3$, (c,d): $s/h = 7.5$, (e,f): $s/h = 7.7$, (g,h): $s/h = 7.9$

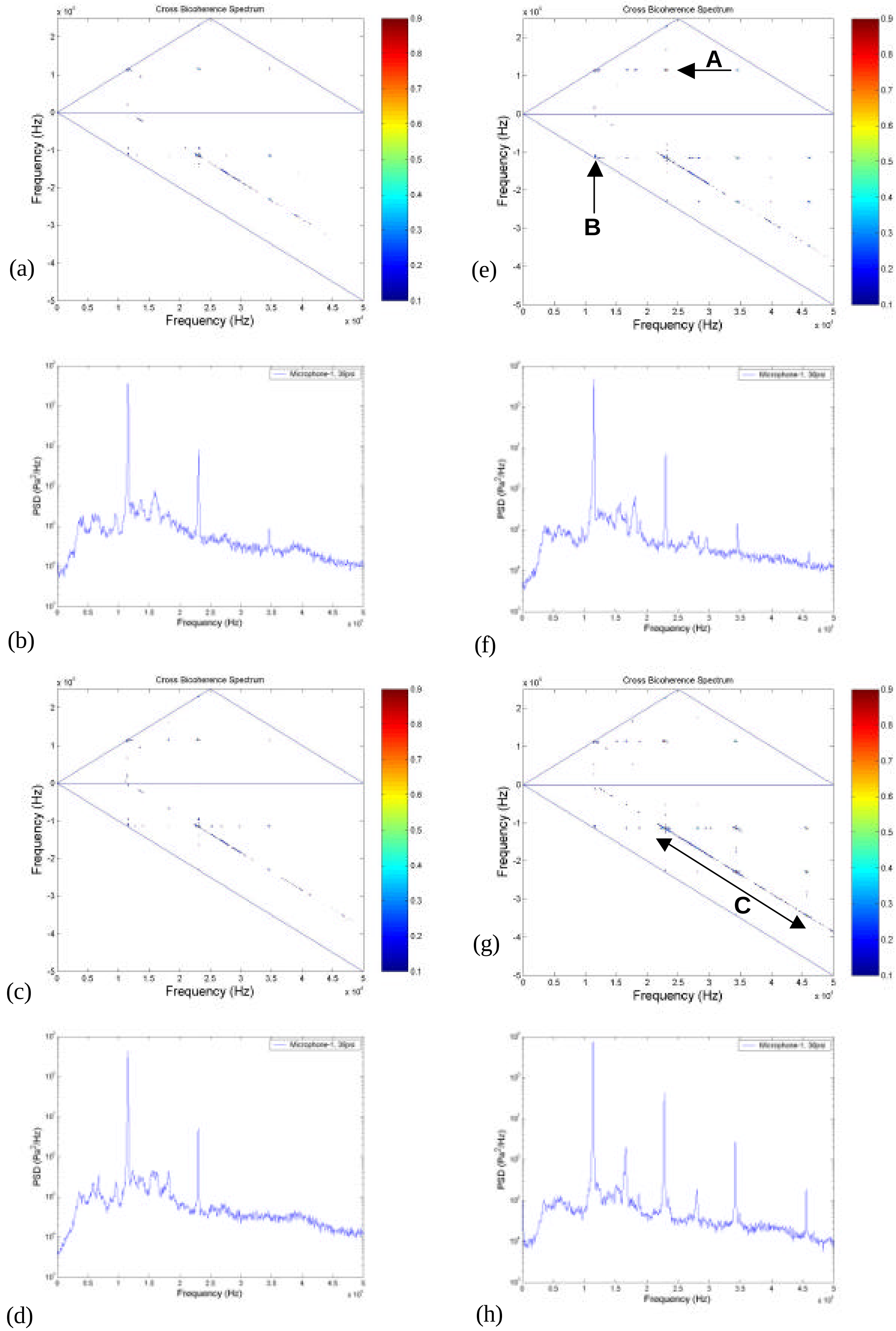


**Figure 6.** Cross-bicoherence and spectra of V-shaped twin jets at $M_j$ = 1.46, for various internozzle spacings. (a,b): *s/h* = 7.3, (c,d): *s/h* = 7.5, (e,f): *s/h* = 7.7, (g,h): *s/h* = 7.9

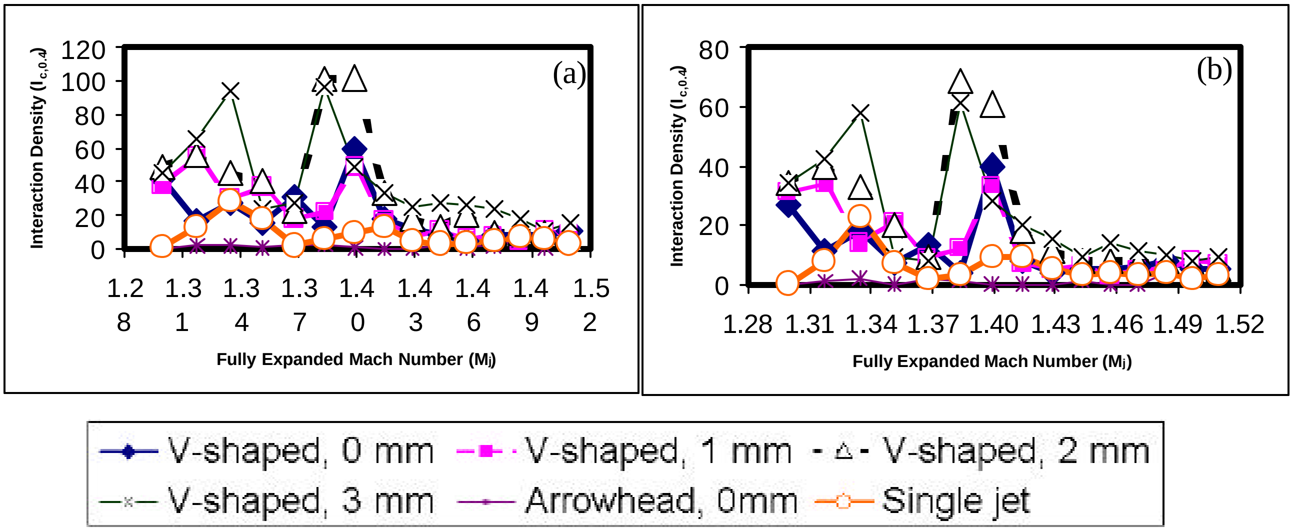


**Figure 7.** Interaction density variation with Mach number. (a) Threshold 0.3. (b) Threshold 0.4.

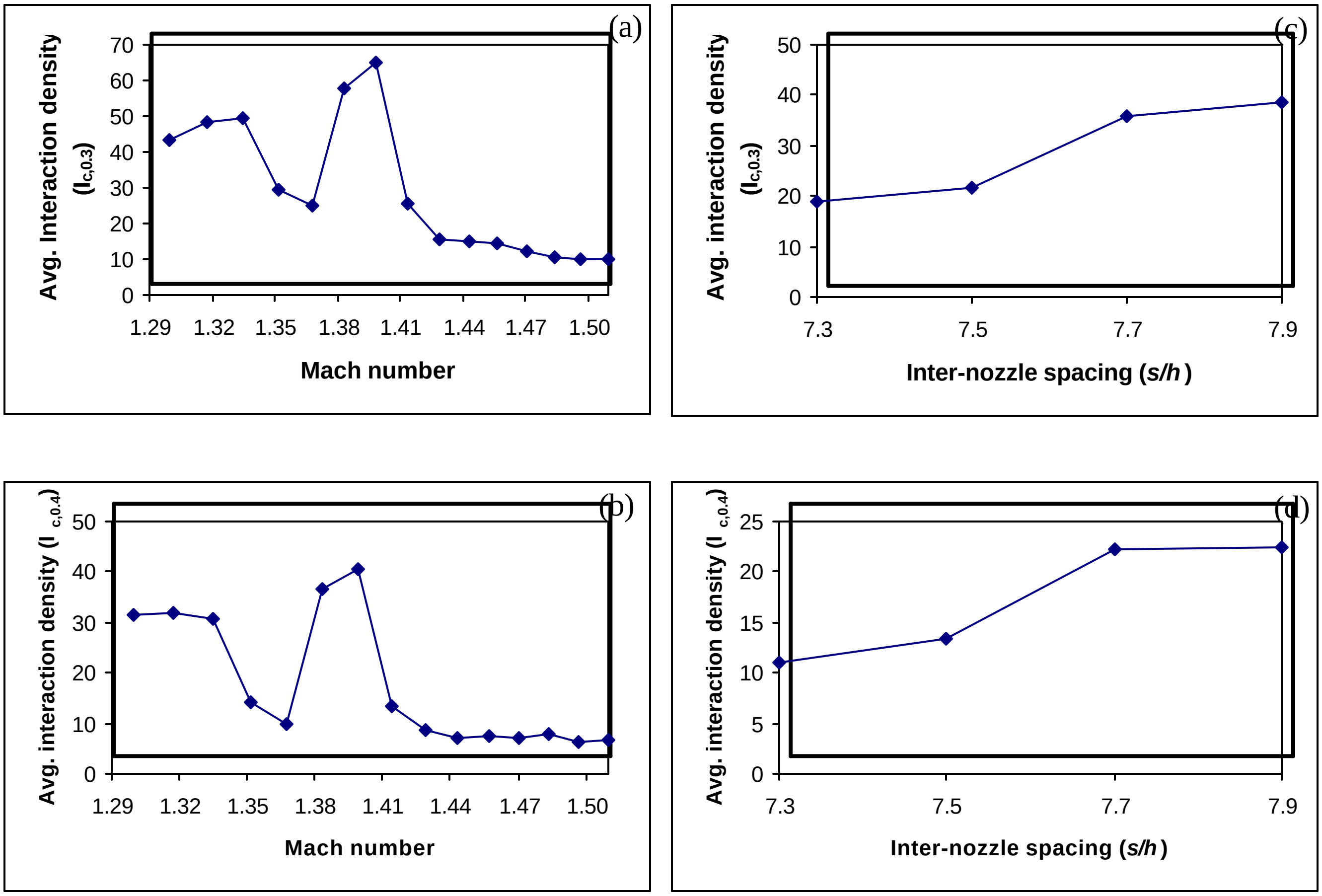


**Figure 8.** Variation of average interaction density with Mach number and inter-nozzle spacing. (a,c) Cross-bicoherence Threshold 0.3, (b,d) Threshold 0.4.